Physics 121
Advanced Laboratory

THE FARADAY EFFECT

A. Introduction:

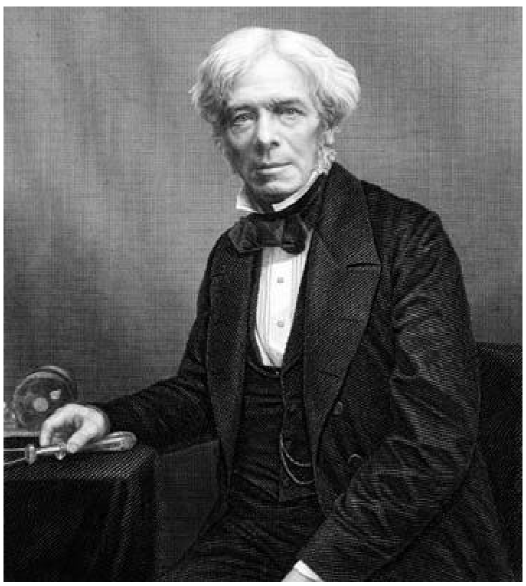
When light propagates along the axis of a transparent rod of length L, the application of an axial magnetic field B to the rod causes a rotation $\Delta\Phi$ of the plane of polarization of the light:

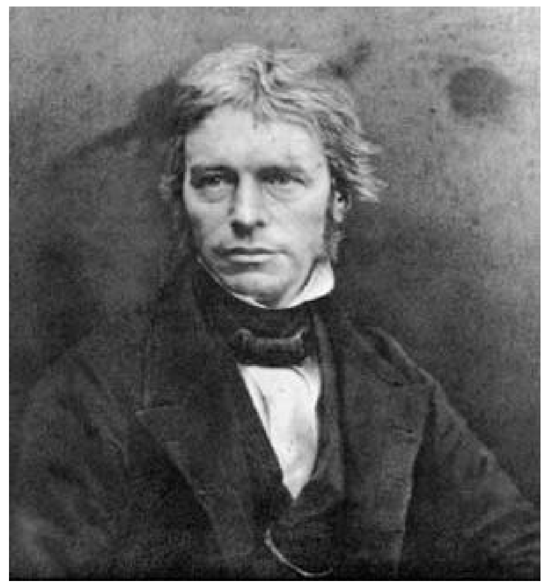
$$\Delta\Phi = VBL \qquad (1)$$

This phenomenon, called the Faraday Effect, occurs in liquids and gasses as well as solids. The Verdet constant V, which measures the strength of the effect, is different in different materials, and varies with the wavelength λ of the light. The Faraday Effect is a consequence of the fact that the magnetic field removes the symmetry for propagation of left-handed and right-handed circularly polarized light, i.e. the index of refraction for light of the two polarizations becomes different in the presence of the field. The relationship between the Verdet constant and the wavelength dependence of the index of refraction depends critically on the nature of the medium, i.e. whether it is semiconducting, diamagnetic, paramagnetic, ferromagnetic, etc.; and hence a correct quantum mechanical calculation requires use of methods from the quantum theory of solids. A simple semi-classical calculation can be given, however, which incorporates much of the essential physics and obtains the right order of magnitude for the effect. This calculation, given below, shows that

$$V = -K(e/2mc^2)\lambda\ (dn/d\lambda) \qquad (2)$$

where e and m are the charge and mass of the electron, c the speed of light and n the index of refraction. (This formula is valid in cgs units for which the Verdet constant has units of radians per Gauss-cm.) The dimensionless constant K measures the deviation of the Verdet constant from the value predicted by the simple semi-classical theory; for molecular hydrogen gas K=0.99; but for solids it can be substantially different from unity. Facilities exist in the Advanced Lab to measure the Verdet constant at fixed wavelength, and to determine the index of refraction and its dependence on wavelength, for rods of two different types of glass. This allows verification of both the basic Faraday Effect (Eq. 1) and of its relation to the index of refraction (Eq. 2).





B. Background References

To perform and adequately comprehend this experiment requires knowledge of each of the following items. (Most of the references given are collected either in this manual, or in the Faraday Effect Equipment Manual (FEEM).)

1) Basic knowledge of the physics of the Farady Effect and its relationship to the wavelength dependence of the index of refraction. See Rutledge's Notes on the Faraday Effect; the MIT Lab Manual Section on Faraday Rotation; and Experiment 22, The Faraday Effect, in Preston and Dietz, *The Art of Experimental Physics*. All three are included in this manual. Other treatments can be found in textbooks on optics (e.g. *Optics*, by K.D. Möller) or magneto-optics.

2) Familiarity with the basic instrumentation for measuring the Faraday rotation. For a d.c. measurement, this includes a diode laser, a solenoid powered by a d.c. power supply, polarizers, and a photodiode output to a d.c. voltmeter for detecting the transmitted light. It also includes an ammeter and a Gaussmeter for measuring the current through the coil and the resulting magnetic field. Equipment manuals are included in FEEM. The basic expermimental setup is similar, although not precisely identical, to that discussed in Expt. 22 of Preston and Dietz.

For an a.c. measurement, an a.c. function generator replaces the d.c. power supply; and a lock-in amplifier replaces the dc voltmeter. A brief discussion on the use of a lock-in amplifier is included in these notes; and further discussion can be found in the Reference Manual Regarding the Use of Lock-in Amplifiers, which can be found in the laboratory. An oscilloscope should also be used in conjunction with the lock-in, to facilitate the measurement.

3) Familiarity with the equipment and techniques utilized for measuring the index of refraction. The manual on use of the Spencer Spectrometer is included in this manual. There are brief discussions on the method of minimum deviation in the Spencer Manual, in the MIT Manual and in Preston and Dietz. You may need to review geometric optics.

C. Experimental Procedure

1. D.C. Measurement of the Faraday Effect

a. Experimental Configuration

The basic experimental setup is shown in Fig. 1.

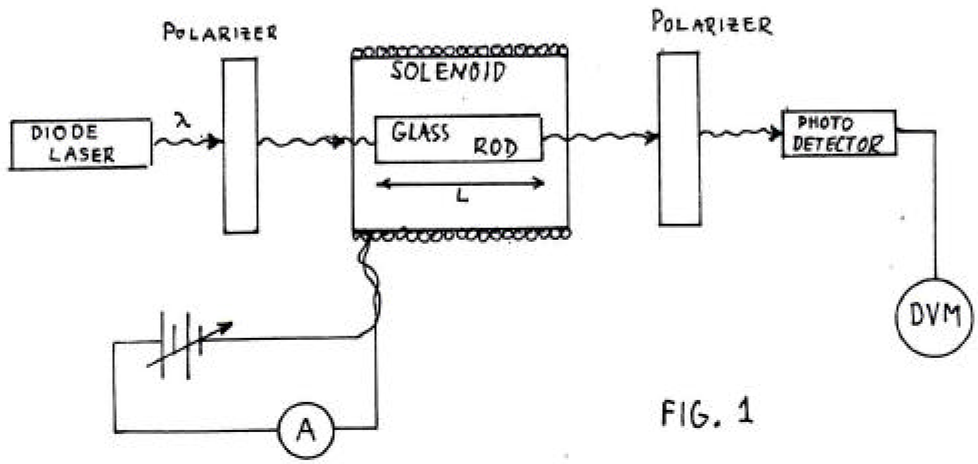
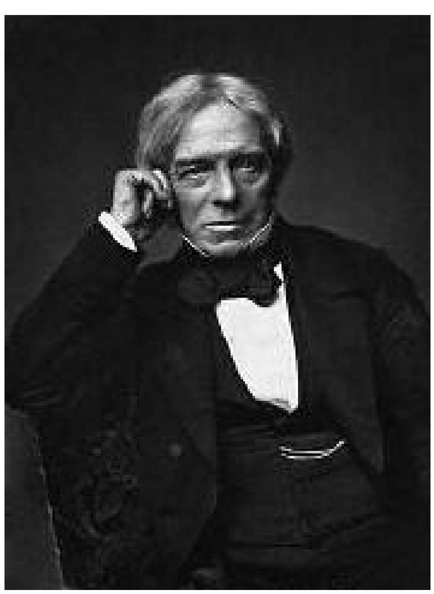
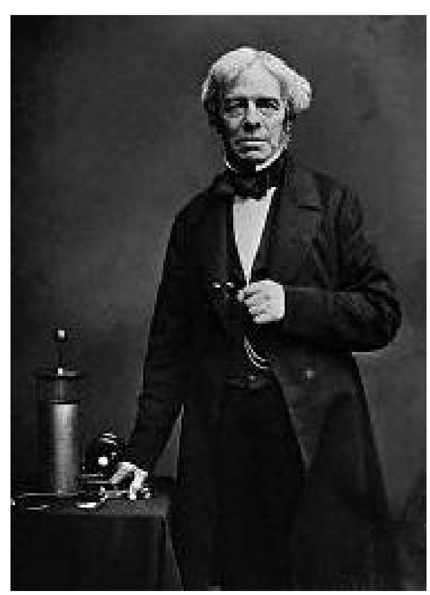


FIG. 1

Monochromatic light of wavelength λ from a diode laser passes through a linear polarizer, and then impinges on a glass rod of length L. This rod is inserted along the axis of a solenoid, which when powered by a d.c. power supply creates a longitudinal magnetic field B(z) in the rod, causing a Faraday rotation $\Delta\Phi$ of the plane of polarization of the light. The light emerging from the solenoid passes through a second polarizer and is then detected by a photodiode, whose d.c. output is read by a digital voltmeter. Adjustment of the relative settings of the polarizers before and after application of the magnetic field allows a determination of $\Delta\Phi$ in the glass; use of Eq. 1 then allows determination of the Verdet constant V at the wavelength of the laser.





C. 1. b. D.C. Measurement Procedure:

1) Two glass rods (Schott Glass SF-57 and SF-58) are available for the experiment. The ends of the rod, and the surfaces of the prisms used in determining the refractive index, are highly polished surfaces; scratches and nicks and fingerprints can cause internal reflection and diffraction of the beam, which can degrade the resolution of the experiment. Extreme care should be taken when handling the rods and prisms: do not mishandle them so as to create nicks, scrapes or breaks, and do not touch the polished surfaces with your fingers. A simple procedure for cleaning the polished surfaces of the rods and prisms -is given below. [Warning: Cleaning of the plastic polarizers should be done with a jet of compressed air or a soft piece of cloth; do not wipe the polarizers with acetone or alcohol!!]

2) A first step should be to calibrate the magnetic field in the coil as a function of current. We note that currents of the order of an amp are required in the experiment; but such currents can cause heating of the coil. This heating will increase the load resistance and can cause the current to vary. Do not operate with currents above 3 amps; and do not pass currents of this magnitude (1-3amps) through the coil for extended periods of time. (Monitor the coil with your fingers to ensure that it does not become too hot to the touch.) To calibrate the coil, use the axial Gaussmeter probe in the d.c. mode. Is the field a linear function of the current? How homogeneous is the field along the axis of the coil? How will inhomogeneity affect the measurement? How can you correct for it?

3) With the diode laser operating and with the glass rod in place, measure the output $S_{B=0}(\Phi)$ of the photodiode at zero magnetic field as a function of the relative polarization angle between the polarizers. What functional form should the output obey? Does it? How stable is the laser output? How will this affect your measurement, and what can you do about it? [Warning: As with any laser, you must be careful not to look directly in to the beam, as this can cause damage to your eye!!] [Warning: The photodiode detector will saturate when too much light is incident on it; this can harm the detector. Furthermore, when operating the photodiode close to saturation, the output will be non-linear.]

4) Finally, determine the Verdet constant by measuring the photodiode output $S_B(\phi)$ at various magnetic fields. Think through how best to utilize the polarizers to obtain maximum sensitivity in the determination of the Faraday rotation $\Delta\Phi$.

5) Make appropriate use of curve fitting and statistical analysis of the data using the software available in the Physics Department Computer Lab (e.g. SigmaPlot). Identify potential sources of error in the measurement, and estimate their effect on the measurement.

C.2. A.C. Measurement of the Faraday Effect

a. Experimental Configuration

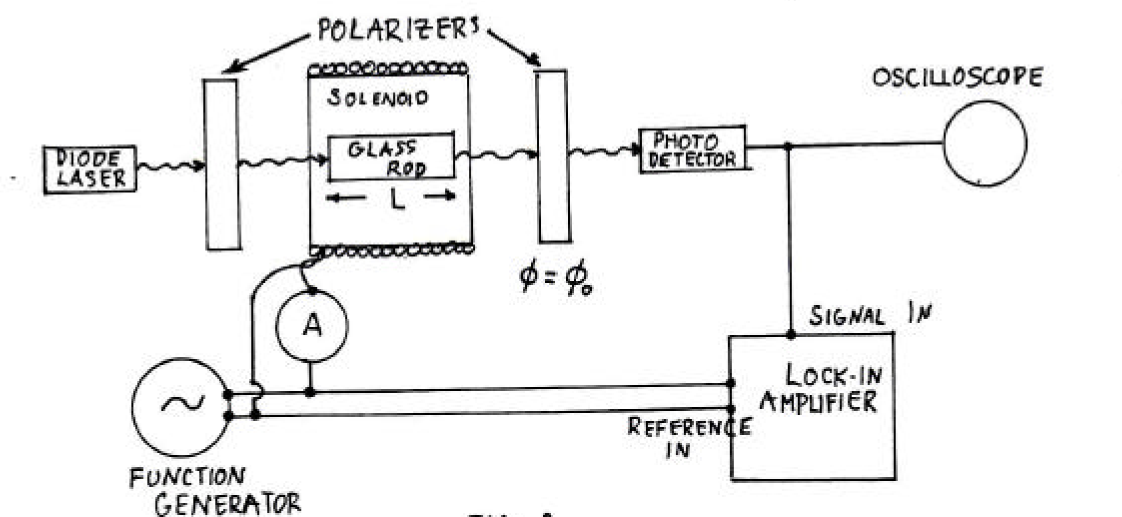


FIG. 2

Use of an a.c. technique for measuring the Faraday Effect has two advantages: it allows for use of currents in the solenoid whose magnitude is much smaller than in the d.c. case, and hence circumvents heating of the coil; and it allows for the use of a lock-in amplifier, with its high inherent signal-to-noise ratio capability. (It is also an excellent example of use of a lock-in.) The basic setup for such an a.c. measurement is shown in Fig. 2. An a.c. sinewave generator is used instead of the d.c. power supply to energize the solenoid, giving rise to an oscillatory magnetic field $B\cos(\omega_0 t)$; the current is read with an a.c. ammeter. The output of the photodetector is input to a lock-in amplifier, whose reference channel is driven at the same frequency as the solenoid by the low output of the function generator. The second polarizer is fixed at some angle ϕ_0. The magnetic field in the coil is small, and hence the changes in the polarization angle arising from the Faraday Effect are small, and oscillatory. The output of the photodiode then has the form

$$S(t) = S_{B=0}(\phi_0 + VLB\cos(\omega_0 t + \delta)) \tag{3}$$

Here, δ is the phase difference between the sinusoidal component of the signal and the signal driving the reference channel. The lock-in acts as a Fourier averager, which reads the root-mean-square magnitude of the component at the driving frequency ω_0. All components of the signal (including the d.c. component, higher harmonics, and noise) at other frequencies than the driving frequency ω_0

are averaged to zero by the lock-in. Expansion of Eq. 3 in a power series shows that the measured signal is

$$S(\omega_0) = (\partial S_{B=0}(\phi_0)/\partial\phi) VLB_{rms} \tag{4}$$

C. 2. b. A.C. Measurement Procedure:

1) As for the d.c. case, calibrate the a.c. magnetic field in the coil as a function of current, and check the axial inhomogeneity. Does the Gaussmeter measure the peak amplitude or the r.m.s. amplitude? (Determine this by comparing the B(I) curve to the d.c. case; the ammeter reads r.m.s.)

2) Use a low frequency (less than 60 Hz) to drive the coil; the photodetector loses gain at higher frequencies. (In fact, it is a good idea to do the complete measurement at several frequencies, to establish its frequency dependence.) Set the current in the range 10-100mamp. Choose a value of ϕ_0 to obtain maximum sensitivity, and set the second polarizer correspondingly.

3) Drive the lock-in reference with the low output of the oscillator; and attach the output of the photodetector to the signal channel input of the lock-in. Since the lock-in is phase sensitive, and since the signal can be phase-shifted relative to the reference by reactance in the input and output circuits, you will have to determine the phase of the signal, which is the phase where the lock-in output is a maximum. (Why is it more sensitive to determine the phase at which the lock-in output vanishes, and then change phase by 90°?)

4) You will have to measure the angular dependence of $S_{B=0}(\phi)$; this can be done simultaneously with the measurement of $S(\omega_0)$, or immediately before or after the a.c. measurement.

5) As for the d.c. case, use of curve fitting and statistical analysis can improve the measurement. Does the a.c. result agree with the d.c. result?

C. 3. Determination of the Wavelength Dependence of the Index of Refraction

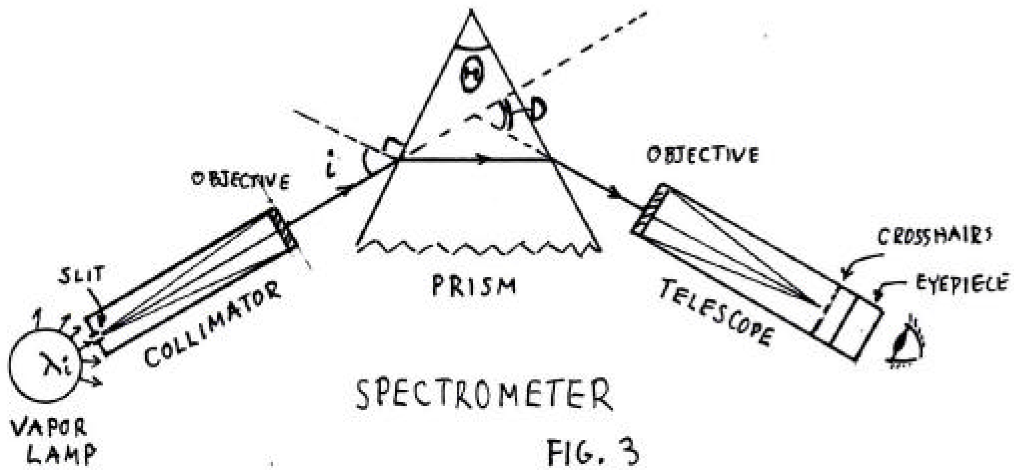


FIG. 3

a. Basic Method

To measure the index of refraction of a prism, light of known wavelength is made incident on a slit S of a collimator C, then traverses a prism P of prism angle Θ, and is then viewed through a telescope T as shown in Fig. 3. The prism and telescope are rotated until the minimum deviation angle D_{min} is determined. The index of refraction is then obtained from the formula

$$n = \sin((\Theta + D_{min})/2) / \sin(\Theta/2) \qquad (5)$$

(Prove this.) To determine the wave length dependence, and its derivative $\partial n/\partial\lambda$, several wavelengths must be used; this is accomplished in the lab by using as a source the different spectral lines emitted in elemental vapor lamps (e.g. a Hg vapor lamp). The wavelength dependence $n(\lambda)$ for most materials can be fit by a two-parameter function (Cauchy's Formula):

$$n(\lambda) = A + (B/\lambda^2) \qquad (6)$$

The constants A and B can be determined by curve fitting the data to Eq. 6; then $\partial n/\partial\lambda$ can be determined at any given frequency by differentiation.

C. 3. b. Procedure for Determining the Index of Refraction

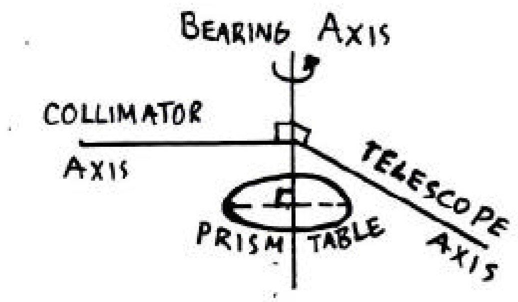


FIG. 4

1) Spectrometer Alignment: The spectrometer must first be adjusted so that the eyepiece is focused on the crosshairs, the crosshairs are at the focal point of the telescope's objective, and the telescope axis, collimator axis and prism table are all perpendicular to the bearing (rotation) axis of the spectrometer (Fig. 4). The full procedure, using the method of autocollimation, is described in the manual for the Spencer Spectrometer; we give a shortened description here.

In the method of autocollimation, light rays incident through the hole in the Gaussian eyepiece illuminate the cross hairs, travel down the axis of the telescope, and then are reflected back into the telescope by a mirror to produce an image at the focal point of the telescope objective. A three-sided mirrored prism is available in the laboratrory for this purpose. (Do not touch the aluminized surfaces!) For a light source, place a white sheet of paper below the eyepiece, and with the hole in the eyepice aimed at the paper, shine a spot lamp onto the paper in an otherwise darkened room.

The first step is to make sure the eyepiece is focussed on the crosshairs. At present (1993) the crosshairs will be in focus very close to the end of the eyepiece; i.e. the focal point of the eyepiece is very close to its end. The second step is to ensure that the crosshairs are at the focal point of the objective; this latter condition is satisfied when the reflected image is in focus at the same plane as the crosshairs (Fig. 5).

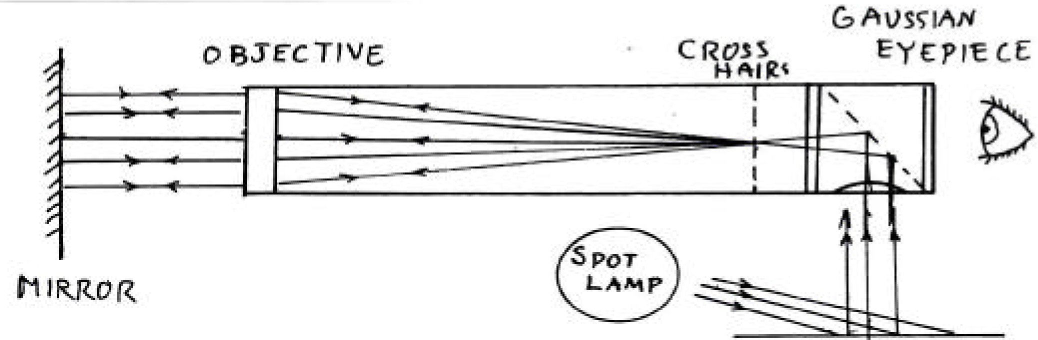
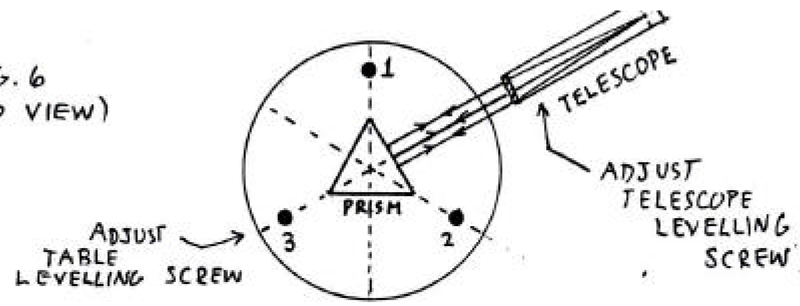


FIG. 5

The adjustment of the telescope axis and prism table are coupled. Center the mirrorred prism on the prism table as shown in Fig. 6. Examine the separation between the horizontal crosshair and its image. Correct half the separation using the telescope leveling screw and half using the prism table levelling screw opposite the face in question (3 in Fig. 6). Rotate the table and do this sequentially for each face in turn, continuing to repeat the measurement until the crosshair and its image coincide on all three faces.

FIG. 6
(TOP VIEW)



With the telescope and prism table normal to the bearing axis, the collimator can now be adjusted. Remove the mirrored prism, energize one of the vapor lamps, and shine it on the slit of the collimator. Move the telescope to look directly into the collimator. Twist the slit to the horizontal position. Focus the light at the crosshairs by adjusting the slit distance; this effectively ensures that the focal point of the collimator's objective is at the slit. Next move the collimator levelling screw to make the light be coincident with the horizontal crosshair. The collimator axis is now normal the bearing axis; so you can rotate the slit back to the vertical position and refocus.

2) Determining the prism angle:

Two prisms exist in the lab corresponding to the two materials measured in the Faraday Rotation experiment. One side of the prism is roughened, the other two polished. The prism angle Θ between the two polished faces should be 60°; you can determine it using the method of autocollimation. With all other sources of light off in the darkened room, shine light into the opening in the Gaussian eyepiece as described above. Find the image of the vertical crosshair reflected from one of the polished faces, and bring it into coincidence with the crosshair. Note the reading. Now rotate the prism table, and do the same for the other polished face. The difference in readings is the prism angle.

3) Measuring the Refractive Index: Method A

First, given that the refractive indices of the SF-57 and 58 glasses are large (1.8-2.0) for the wavelengths utilized, what will the corresponding values of D_{min} be? With this as foreknowledge, center the prism on the table and align the prism and telescope with respect to the collimated beam approximately as expected. Rotate the prism table and telescope by small ammounts until the spectral line is in view. Continue by smaller amounts until the condition of minimum deviation is fulfilled. (How do you recognize this condition?) Note the angle. Now remove the prism, and rotate the telescope until the vertical light is coincident with the crosshairs. The measured change in angle should equal D_{min}. (Why?) Repeat the measurement with the light incident on the other polished face, and the telescope making the opposite angle ($-D_{min}$) with respect to the incident beam.

4) Measuring the Refractive Index: Method B

Find the condition of minimum deviation for the light incident on face BA as shown in Fig. 7. Then rotate until you find the condition for light incident on face CA. Determine the difference $\Delta\theta$ in angular readings. (In doing this be careful to monitor the change in angle due to rotating the telescope, and due to rotating the prism table; and note whether the angle passes through 360°.) Considering the geometry of Fig. 7, if the prism angle Θ is exactly equal to 60°, by what angle R_p must you rotate the prism to go from minimum deviation on the one face to minimum deviation on the other? (Use the relationships between D_{min} and angle of incidence i shown in the Spencer Manual to determine R_p.) By what angle R_t must you rotate the telescope? Do the two rotations R_p and R_t add or subtract to determine $\Delta\theta$? How does the result compare with that of method A?

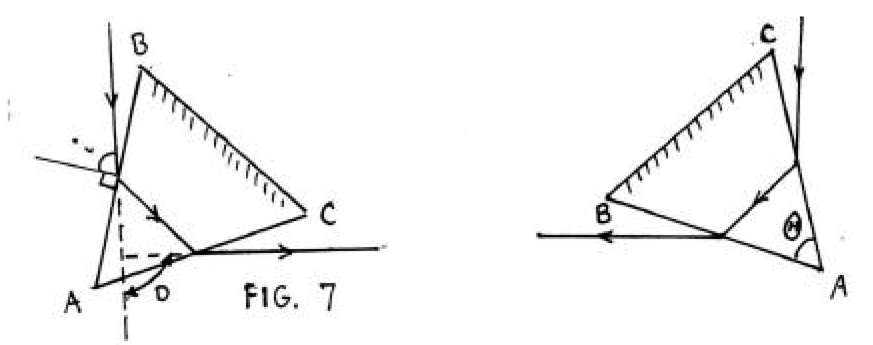


FIG. 7

5) Data Analysis:

Fit the data for several wavelengths to Eq. 6 using the software packages in the UCI Physics Department Computer Lab. Clearly, the quality of the fits will be improved if you use more spectral lines. There are several sources in the lab; values of the wavelength can be established from standard tables (e.g. the table "Persistent Lines of the Elements" in the *CRC Handbook of Chemistry and Physics*). Alternatively, as an advanced project, a spectrometer exists in the Advanced Lab to directly measure the wavelenghts. Finally, relate the measurement of $n(\lambda)$ to the measured Faraday Rotation, determining how far the constant K differs from unity.